Math 3303 Final Exam

Question 1 14 points

Describe two partitions: the first one on all the natural numbers and the second one on all the real numbers. Tell why your description is a partition on the relevant set.

Question 2 12 points

Illustrate the theorem at least twice:

The square of any integer is in [0] or [1] mod 3 and not [2] mod 3.

Question 3 20 points

Short Answers needed:

3A Given that $f(x) = \dfrac{x}{x-3}$ is one-to-one. What is f^{-1} ?

3B Given $f(x) = x^2 + 3$ and $g(x) = 2x + 1$. What is $g \circ f(3)$?

3C Is the set of all one-to-one functions with the operation composition a group?
Support your answer with function facts.

3D Given an example of a relation on two sets that is NOT a function. Explain why it is not.

Question 4 14 points

Why is the set of natural numbers mod 5 a group with the operation addition?
Sketch a Cayley Table and give your explanation in agonizing detail.

Question 5 20 points

Given this set of numbers: $\{ \frac{\pi}{2}, \quad \sqrt{7}, \quad -1+i, \quad 6, \quad 2 \}$

Write each number below and pick the adjectives from the list below that apply to them:

Abundant, Algebraic, Complex, Composite, Deficient, Irrational, Perfect, Prime, Quaternion, Rational, Transcendental

Question 6 10 points

Illustrate the theorem:

If n is an integer, not divisible by 2 or 3, then $n^2 + 23$ is divisible by 24.

Extra Credit Question 5 points

If $ac \equiv_m bc$ then $a \equiv_{m/d} b$ where $d = (c, m)$. $a, b, c,$ and d are natural numbers, d is greater than 1 and both m and c are multiples of d.